

Calculus Warm Up #11-4

Evaluate:

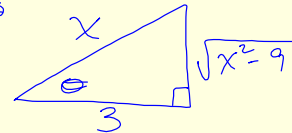
1. $\int \frac{x}{\sqrt{x^2 + 9}} dx$

2. $\int \frac{x^3 - 4x + 2}{x - 3} dx$

HW Questions: p. 526

17. $\int \frac{1}{\sqrt{x^2 - 9}} dx$

$$x = 3 \sec \theta \rightarrow \sec \theta = \frac{x}{3}$$
$$dx = 3 \sec \theta \tan \theta d\theta$$



$$\int \frac{1 \cdot 3 \sec \theta \tan \theta d\theta}{3 \tan \theta}$$
$$\ln \left| \frac{x + \sqrt{x^2 - 9}}{3} \right| + C$$

$$3 \tan \theta = \sqrt{x^2 - 9}$$

19. $\int_0^{\sqrt{3}/2} \frac{t^2}{(1 - t^2)^{3/2}} dt$

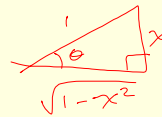
$$\ln |x + \sqrt{x^2 - 9}| - \ln 3 + C$$

21. $\int \frac{\sqrt{1-x^2}}{x^4} dx$

$\sin \theta = x$

$dx = \cos \theta d\theta$

$\cos \theta = \sqrt{1-x^2}$



$\int \cos \theta \cdot \cos \theta d\theta$

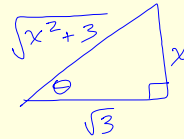
$\int \frac{\cos^2 \theta}{\sin^2 \theta} \cdot \csc^2 \theta d\theta$

$\int \frac{\cos^2 \theta}{\sin^2 \theta} \cdot \csc^2 \theta d\theta$

25. $\int \frac{x}{(x^2+3)^{3/2}} dx$

$x = \sqrt{3} \tan \theta$

$dx = \sqrt{3} \sec^2 \theta d\theta$



$\int \frac{x}{(\sqrt{x^2+3})^3} dx$

$\sqrt{3} \sec \theta = \sqrt{x^2+3}$

$\int \frac{\sqrt{3} \tan \theta \cdot \sqrt{3} \sec^2 \theta d\theta}{(\sqrt{3} \sec \theta)^3}$

$\int \frac{\cancel{3} \tan \theta \cancel{\sec^2 \theta}}{\cancel{3} \sqrt{3} \sec \theta} d\theta$

$\frac{1}{\sqrt{3}} \int \frac{\tan \theta}{\sec \theta} d\theta$

$\frac{1}{\sqrt{3}} \int \frac{\sin \theta}{\cos \theta} \cos \theta d\theta$

$\frac{1}{\sqrt{3}} \int \sin \theta d\theta$

27. $\frac{1}{2} \int 2e^{2x} \sqrt{1+e^{2x}} dx$

let $u = 1 + e^{2x}$

$du = 2e^{2x} dx$

29. $\int (x+1) \sqrt{x^2+2x+2} dx$

31. $\int \frac{x}{\sqrt{x^2 + 4x + 8}} dx$

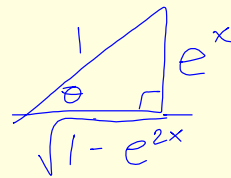
$$\begin{aligned} x^2 + 4x + 4 + 4 \\ (x+2)^2 + 2^2 \\ x+2 = 2 \tan \theta \end{aligned}$$

33. $\int e^x \sqrt{1 - e^{2x}} dx$

$$\begin{aligned} \text{let } e^x &= \sin \theta \\ e^x dx &= \cos \theta d\theta \end{aligned}$$

$$\int (\cos \theta d\theta) \cdot \cos \theta$$

$$\int \cos^2 \theta d\theta$$



$$\cos \theta = \sqrt{1 - e^{2x}}$$

Classwork: p. 526, # 5 - 8

Answers:

5. $\ln |x + \sqrt{x^2 - 4}| + C$

6. $\sqrt{x^2 - 4} - 2 \operatorname{arccsc}\left(\frac{x}{2}\right) + C$

7. $\frac{16}{3}(x^2 - 4)^{3/2} + \frac{16}{5}(x^2 - 4)^{5/2} + C$

8. $4\sqrt{x^2 - 4} + \frac{1}{3}(x^2 - 4)^{3/2} + C$

9.5 Partial Fractions

for $\int \frac{f(x)}{g(x)} dx$

When degree of top $<$ degree of the bottom
and the bottom is factorable

When top $>$ bottom, do the division!

$$\int \frac{x+7}{x^2-x-6} dx$$

Rewrite:

$$\frac{x+7}{(x-3)(x+2)} = \frac{A(x+2)}{(x-3)(x+2)} + \frac{B(x-3)}{(x+2)(x-3)}$$

Solve Basic Equation for A & B: $x+7 = A(x+2) + B(x-3)$

$$\text{Let } x = -2 \rightarrow -2+7 = A(-2+2) + B(-2-3)$$

$$5 = 0 + B(-5) \rightarrow B = -1$$

$$x = 3 \rightarrow 10 = 5A$$

$$A = 2$$

Integrate:

$$\begin{aligned} \int \frac{x+7}{x^2-x-6} dx &= \int \frac{2}{x-3} dx - \int \frac{1}{x+2} dx \\ &= 2 \ln|x-3| - \ln|x+2| + C \\ &= \ln \left| \frac{(x-3)^2}{x+2} \right| + C \end{aligned}$$

Denominators with repeat factors:

$$\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$$

Rewrite:

$$\frac{5x^2 + 20x + 6}{x(x+1)^2} = \frac{A(x+1)^2}{x(x+1)^2} + \frac{Bx(x+1)}{(x+1)^2x} + \frac{Cx}{(x+1)^2x}$$

Solve Basic Equation for A, B & C:

$$5x^2 + 20x + 6 = A(x+1)^2 + Bx(x+1) + Cx$$

$$\text{Let } x=0 \quad (6=A)$$

$$x=-1 \quad 5-20+6=C(-1) \rightarrow (C=9)$$

$$x=1 \quad 31=6(4)+B(1)(2)+9(1) \\ (B=-1)$$

Denominators with repeat factors:

Integrate:

$$\begin{aligned} \int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx &= \int \left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \right) dx \\ &= \int \left(\frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2} \right) dx \\ &= 6 \ln|x| - \ln|x+1| - \frac{9}{x+1} + C \end{aligned}$$

Denominators with repeat factors:

$$\int \frac{5}{x^3(x+2)} dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+2}$$

Each power of the repeat factor needs its own partial fraction.

Quadratic factors:

$$\int \frac{3x^2 + 4x + 4}{x^3 + 4x} dx$$

Rewrite:

$$\frac{3x^2 + 4x + 4}{x(x^2 + 4)} = \frac{A(x^2 + 4)}{x(x^2 + 4)} + \frac{(Bx + C)x}{(x^2 + 4)x}$$

Solve Basic Equation:

$$3x^2 + 4x + 4 = A(x^2 + 4) + x(Bx + C)$$

$$\text{Let } x=0 \quad 4 = 4A \rightarrow A=1$$

$$x=1 \quad 11 = 1(5) + 1(B+C)$$

$$6 = B+C$$

$$x=-1$$

$$3 - 4 + 4 = 1(5) + (-1)(B(-1) + C)$$

$$-2 = B - C$$

Solve System
 $B=2$
 $C=4$

Integrate:

$$\begin{aligned}\int \frac{3x^2 + 4x + 4}{x^3 + 4x} dx &= \int \frac{1}{x} dx + \int \frac{2x + 4}{x^2 + 4} dx \\&= \ln|x| + \int \frac{2x}{x^2 + 4} dx + \int \frac{4}{x^2 + 4} dx \\&= \ln|x| + \ln(x^2 + 4) + 2 \arctan\left(\frac{x}{2}\right) + C \\&= \ln|x^3 + 4x| + 2 \arctan\left(\frac{x}{2}\right) + C\end{aligned}$$

HW: p. 536, # 3 - 11 odd,
15, 27, 31